Published by Institute of Physics Publishing for SISSA
Received: *June 14, 2005*
Revised: *July 25, 2005*
Accepted: *September 7, 2005*
Published: *September 13, 2005*

Lorentz violation in extra dimensions*

Thomas G. Rizzo
Stanford Linear Accelerator Center
2575 Sand Hill Rd., Menlo Park, CA, 94025
E-mail: rizzo@slac.stanford.edu

Abstract: In theories with extra dimensions it is well known that the Lorentz invariance of the $D = 4 + n$-dimensional spacetime is lost due to the compactified nature of the n dimensions leaving invariance only in 4d. In such theories other sources of Lorentz violation may exist associated with the physics that initiated the compactification process at high scales. Here we consider the possibility of capturing some of this physics by analyzing the higher dimensional analog of the model of Colladay and Kostelecky. In that scenario a complete set of Lorentz violating operators arising from spontaneous Lorentz violation, that are not obviously Planck-scale suppressed, are added to the Standard Model action. Here we consider the influence of the analogous set of operators which break Lorentz invariance in 5d within the Universal Extra Dimensions picture. We show that such operators can greatly alter the anticipated Kaluza-Klein(KK) spectra, induce electroweak symmetry breaking at a scale related to the inverse compactification radius, yield sources of parity violation in, e.g. , 4d QED/QCD and result in significant violations of KK-parity conservation produced by fermion Yukawa couplings, thus destabilizing the lightest KK particle. LV in 6d is briefly discussed.

Keywords: Beyond Standard Model, Large Extra Dimensions, Compactification and String Models.

*Work supported in part by the Department of Energy, Contract DE-AC02-76SF00515.

JHEP09(2005)036

Contents

1. Introduction

The possibility that Lorentz invariance(LI) may be violated at high energies in 4d with testable consequences has become a subject of much interest in the past few years [1]. If one considers rotational invariance to be sacrosanct due to the strength of existing experimental constraints, Lorentz violation(LV) must take the form of a breakdown of invariance under boosts. Such a scenario is suggestive of spontaneous LV [2] where in some preferred frame, e.g. , a time-like 4-vector takes on a vacuum expectation value with components $\sim (1, 0, 0, 0)$.

While LV of the type described above have yet to be observed in 4d it is clear that if n extra (flat) compact dimensions exist, perhaps near the TeV scale, they obviously behave differently than do the large dimensions with which we are familiar. Clearly the LI in the $D = 4+n$ dimensional space has been lost through the compactification process; LI suffers further damage if the compactified manifold (here considered to be a torus) is orbifolded. In the usual bottom-up analysis, the D-dimensional action is conventionally written in a completely LI manner with all LV arising from the compactification/orbifolding process in the IR. Thus, in the UV, such a theory apparently maintains LI. While such an approach may be the simplest one to analyze it certainly does not address the larger issue as to how or why the n dimensions have become compact. Some unknown UV physics must have triggered this compactification process making the 4 large dimensions (or, depending on one's viewpoint, the n compactified dimensions) 'special' otherwise we would be living in

$4+n$ dimensions. Thus the true UV physics cannot be completely LI in D-dimensions. It would be interesting to ask if this UV physics has left any signatures for us to find at accelerators that are beginning to probe the TeV energy regime near the compactification scale, R^{-1}. If present, how might such effects be parameterized?

For simplicity let us consider the case of one extra dimension, i.e. , a theory in 5d with the extra dimension compactified as usual on S^1/Z_2 with $R^{-1} \lesssim$ few TeV. If the UV breaking of LI is spontaneous, as in the 4d case discussed above, we can imagine that it was triggered by a 5-vector taking on the vev $\sim (0,0,0,0,1)$ in some frame [2] thus leaving us with 4d LI with the fifth dimension becoming 'special'. Although our approach will not capture any of the detailed dynamics associated with how such a vev was generated, it may be able to probe some of the residual LV physics which could remain accessible at the TeV scale. A framework for this type of analysis already exists for the Standard Model(SM) in 4d, the so-called SM Extension [3]. This framework is particularly appealing for a number of reasons one of which is that it isolates the dominant effects of explicit LV in operators of the lowest possible dimension. In order to have a specific model in which to work in 5d that is closely analogous to this 4d case, we here adapt this particular framework to the 5d version of SM where all fields are in the bulk, i.e. , the Universal Extra Dimensions(UED) [4] scenario, but allowing LV to occur only in the fifth dimension.

As we will see in detail below the existence of LV in 5d induces a number of new effects within the 5d UED framework such as: (i) The KK spectra of the gauge and Higgs bosons as well as those for all of the left- and right-handed fermions can each be rescaled arbitrarily. This can increase the possible confusion between UED and SUSY scenarios at the LHC [4]. (ii) New loop-induced parity violating effects are generated within previously parity conserving sectors of the model, e.g. , in 5d QED/QCD. One signature of this is the generation of anapole moment-type couplings between fermions and gauge bosons. (iii) KK-parity, which is exact even at loop order in ordinary UED, becomes broken through mixings in the fermion KK mass matrices. This results in loop-induced mixing among gauge KK states and a finite lifetime for the Lightest KK Particle(LKKP) which usually considered as a stable dark matter candidate in the UED scenario. (iv) It is possible that the 5d LV operators may be the source of electroweak symmetry breaking.

The outline of this paper is as follows: In section 2 we discuss the adaptation of the LV SM Extension operators to 5d under the assumption that LI is broken only in 5d. Here we also show how the 5d analogs of the 4d CPT violating (as well as LV) operators can be (almost) removed from the action by suitable field redefinitions. We will show that through these redefinitions these operators may induce spontaneous symmetry breaking by generating a negative mass squared for scalar fields. This leads to a potential correlation between the SM Higgs vev and the size of the extra dimension, R^{-1}. In section 3, we analyze how the remaining operators lead to modifications in the KK spectra of the SM gauge, scalar and fermion fields. We show that having a different KK spectrum for left- and right-handed fermions, which is possible now that 5d LI is broken, yields a potential source for parity violation in QED/QCD in 4d. In section 4 we demonstrate that KK-parity violation results from the Yukawa sector of the theory that normally generates masses for the would be fermion zero-modes. This again arises from the field redefinitions employed earlier to remove the analogs of the CPT violation operators. KK-parity violation is shown to lead to a number of new effects such as the instability of the lightest KK-parity odd particle as well as general mixing among the KK-even and KK-odd excitations. A discussion and our conclusions can be found in section 5. The appendix contains a brief discussion of LV in 6d for scalar fields.

2. Lorentz violating operators

Given the field content of the SM, the SM Extension [3] provides for us a relatively short list of the lowest-dimension gauge invariant 4d LV operators which may also be CPT violating. We can easily adapt this list to our purposes and restrict ourselves to those cases where only 5d LI is lost while 4d LI remains. This requirement turns out to be highly restrictive as many of the 4d LV operators do not have 5d analogs which can lead to loss of LI in only 5d. In principle, one can extend this list by including new operators generated by gravity modifications in 5-d as has been done in ref. [5]. However, since we are restricting ourselves to the UED framework with one additional TeV scale extra dimension wherein gravitational effects we will not discuss this possibility here.

Systematically going through the list 4d operators we find a number whose generalizations to 5d cannot satisfy our constraints. Consider for example the LV set of 4d operators

$$\Phi^\dagger(\alpha_{\mu\nu}B^{\mu\nu} + \beta_{\mu\nu}T^a W_a^{\mu\nu})\Phi\,, \tag{2.1}$$

where Φ is the Higgs scalar and $B^{\mu\nu}$ and $W_a^{\mu\nu}$ are the U(1) and SU(2) field strength tensors; $\alpha_{\mu\nu}$ and $\beta_{\mu\nu}$ are sets of numerical coefficients. Generalizing to 5d we immediately obtain

$$\Phi^\dagger(\alpha_{MN}B^{MN} + \beta_{MN}T^a W_a^{MN})\Phi\,. \tag{2.2}$$

We now ask what values of A, B are allowed for the coefficients above without violating 4d LI: if A, B both take on 4d indices then 4d LI will be broken. Similarly, if, e.g. , $A = \mu$ and $B = 5$ then 4d LI is again lost; the last possibility, i.e. , $A = B = 5$, yields zero due to the index antisymmetry. Thus the generalization of operators such as this in 4d to 5d does not yield anything interesting given the assumptions of our analysis. As another example of this, consider the 4d operator

$$\kappa_{\mu\nu}\bar{D}\sigma^{\mu\nu}S\Phi + \text{h.c.}\,, \tag{2.3}$$

where $D(S)$ is an SU$(2)_L$ doublet(singlet) fermion field and $\kappa_{\mu\nu}$ a set of numerical coefficients. In 5d this trivially generalizes to

$$\kappa_{AB}\bar{D}\sigma^{AB}S\Phi + \text{h.c.}\,. \tag{2.4}$$

As in the previous example the various possible choices of A, B yield either LV in 4d or are zero by the antisymmetry of the indices.

Let us now turn to the set of surviving operators. As an example, in 4d, suppressing flavor indices one has the following possible 'kinetic' LV term, e.g. , for an SU$(2)_L$ singlet fermion field, S:

$$\frac{i}{2}(c_S)_{\mu\nu}\bar{S}\gamma^\mu\bar{D}^\nu S\,, \tag{2.5}$$

where $\bar{D}^\nu$ is a covariant derivative acting in both directions and the c_S are a set of dimensionless numerical coefficients; we expect these coefficients to be very small in 4d. Here we wish to generalize this operator to 5d, i.e. ,

$$\frac{i}{2}(c_S)_{AB}\bar{S}\Gamma^A\bar{D}^B S\,, \tag{2.6}$$

where $\Gamma^\mu = \gamma^\mu$, $\Gamma^5 = i\gamma_5$ and only keep terms which are LV in 5d but not in 4d. Clearly, given the experience of the examples above, we can only choose $A = B = 5$ and taking $k_S = (c_S)_{55}$ this term becomes

$$\frac{k_S}{2}[\bar{S}\gamma_5 D_5 S - (D_5\bar{S})\gamma_5 S]\,. \tag{2.7}$$

Perhaps, more naturally, in 5d we might anticipate that $k_S = O(1)$ since LI in 4d remains unbroken. Of course we may expect a similar term to be present for an $\mathrm{SU}(2)_L$ doublet as in the singlet case described above, with $k_S \to k_D$, but which need not be the same value. We will assume for simplicity that these 5d fermion terms are flavor-diagonal in what follows and denote their set of coefficients more generically by k_Ψ. Going through and attempting to generalize the remaining set of SM Extension 4d operators we find that only very few of them satisfy our 5d requirements; in addition to the 'kinetic'-like operator above for fermions we find the following possibilities:

$$\begin{aligned}
\frac{1}{4}k_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} &\to \frac{\lambda}{4}\Big(F_{\mu 5}F^{\mu 5} + F_{5\mu}F^{5\mu}\Big)\\
k'_{\mu\nu}(D^\mu\Phi)^\dagger(D^\nu\Phi) &\to -k_\Phi(D_5\Phi)^\dagger(D_5\Phi)\\
a_\mu\bar{f}\gamma^\mu f &\to i\alpha\bar{f}\gamma_5 f\\
i(k_\phi)^\mu\Phi^\dagger D_\mu\Phi + \text{h.c.} &\to ih_\Phi\Phi^\dagger D_5\Phi + \text{h.c.}\,,
\end{aligned} \tag{2.8}$$

where the 'mapping' from 4d to 5d is shown explicitly. In the equation above, Φ represents the Higgs doublet as before and, correspondingly, F represents any of the SM field strength tensors. Likewise, f is either an $\mathrm{SU}(2)_L$ doublet, D, or singlet, S, fermion. Note that the first two operators above lead to modifications of gauge and Higgs kinetic terms as was the case for the fermionic operator discussed previously. Also note that in 4d the last two operators in the equation above are CPT violating; we note that in 5d their coefficients must be Z_2-odd in a manner similar to that of any 5d bulk fermion mass term. The parameters $\lambda, k_{\Phi,\Psi}$ are dimensionless quantities which we might expect to be of order unity; they must be highly suppressed quantities in the usual 4d SM Extension. On the otherhand the coefficients α and h_Φ have the dimensions of mass and might most naturally be of order $\sim R^{-1}$. While it is possible that higher dimensional operators may also be present in addition to the ones considered above these are likely to be Planck suppressed and can be safely ignored in the analysis below.

The LV operators that we have found above are for a 5d scenario. It would be interesting to consider how this operator set would be modified by going to even higher dimensions, e.g. , 6d. Here we could imagine that not only is LI of the type that we have been discussing violated in the higher dimensional action but also rotational invariance in the extra dimensions may be lost leading to very interesting new physics. Such possibilities will be considered briefly in the appendix and in detail elsewhere.

It was noted in ref. [3] that some of these CPT violating operators can be eliminated from the action in 4d by suitable field redefinitions. This remains especially true here in 5d (since we are only considering LV in the one extra dimension) but with interesting consequences since these field redefinitions will now depend on the co-ordinate of the extra fifth dimension. In a way, this field redefinition resums the effects of these operators in a non-perturbative way into the fields themselvess. Leaving these operators in the action, one would obtain similar effects order by order in the perturbation theory in the new LV parameters. The field redefinition simplifies the action and allows us to provide a non-perturbative treatment of these operators. As an example, consider the scalar part of the action including the two relevant LV terms above:

$$\int d^4x\ dy\ \left[(D_A\Phi)^\dagger\left(D^A\Phi\right) - V\left(\Phi^\dagger\Phi\right) - k_\Phi\left(D_5\Phi\right)^\dagger\left(D_5\Phi\right) + ih_\Phi\left(\Phi^\dagger D_5\Phi - \Phi D_5\Phi^\dagger\right)\right], \tag{2.9}$$

where we use y as the co-ordinate for the extra dimension and V is the usual scalar potential. If we now make a field redefinition

$$\Phi \to e^{i\Sigma_\Phi y}\Phi\,, \tag{2.10}$$

where Σ is a parameter whose value we choose to be (recall h_Φ is Z_2-odd)

$$\Sigma_\Phi = -\frac{h_\Phi}{1+k_\Phi}\,, \tag{2.11}$$

then the action becomes

$$\int d^4x\ dy\ \left[(D_A\Phi)^\dagger\left(D^A\Phi\right) - V\left(\Phi^\dagger\Phi\right) + \Sigma_\Phi^2\left(1+k_\Phi\right)\Phi^\dagger\Phi - k_\Phi\left(D_5\Phi\right)^\dagger\left(D_5\Phi\right)\right], \tag{2.12}$$

thus eliminating the 'CPT violating' term but now introducing a new contribution to the scalar potential. Note that although the parameter Σ_Φ must be Z_2-odd to maintain the original symmetry only its square now appears in the action. Though the 'CPT' violating operator no longer appears its effects will remain important as we will see below.

Interestingly it is possible that this new quadratic term could produce a negative mass squared for the scalar Higgs field thus generating spontaneous symmetry breaking in the potential. Since we imagine that most naturally $\Sigma_\Phi \sim R^{-1}$ in magnitude this would tell us that the weak scale is linked to the size of the compactification scale up to order one corrections. Note that our field redfinition is consistent with the original Z_2 symmetry. Also note that additional kinetic term proportional to k_ϕ can induce tachyonic states, i.e. , instabilities and /or causality violations, unless the value of this parameter is restricted to be ≥ -1. Such effects were observed in the 4d case when LV was present [6].

A similar field redefinition trick can also be applied in the fermion sector to rid ourselves of the 'CPT violating' term. Let Ψ be any 5d fermion field; the relevant action is then

$$\int d^4x\ dy\ \left[\frac{i}{2}\bar\Psi\Gamma^A\bar D_A\Psi - \frac{1}{2}k_\Psi\bar\Psi\gamma_5\bar D_5\Psi - i\alpha\bar\Psi\gamma_5\Psi\right], \tag{2.13}$$

where we have neglected any bulk mass terms as is standard in UED. Now we make the field redefinition (which maintains the original Z_2 symmetry)

$$\Psi \to e^{i\Sigma_\Psi y}\Psi\,, \tag{2.14}$$

with

$$\Sigma_\Psi = \frac{\alpha}{1+k_\Psi}\,, \tag{2.15}$$

and the 'CPT violating' term is eliminated leaving the action

$$\int d^4x\ dy\ \left[\frac{i}{2}\bar{\Psi}\Gamma^A\bar{D}_A\Psi - \frac{1}{2}k_\Psi\bar{\Psi}\gamma_5\bar{D}_5\Psi\right], \tag{2.16}$$

this time with no additional terms. As in the case above we note that the coefficient Σ_Ψ must be Z_2-odd. Thus out of the five possible LV structures in 5d we can eliminate two of them by field redefinitions; as we will see these redefinitions will return to haunt us later on. Note that the remaining LV terms are all modifications to kinetic terms and are all dimension-5, i.e. , they are of the same dimension as are the usual SM-like terms in the 5d action.

3. Influence of LV terms: KK spectrum

The three remaining LV terms have a common feature: they are modifications of the 5d kinetic terms for fermions, Higgs bosons and gauge fields. They are analogous to (but not the same as) the addition of brane kinetic terms in the action [7] and will produce similar effects as we will now see. We remind the reader that the LV contributions discussed below occur at the tree level while the somewhat analogous effects observed in the UED occur at loop order.

For simplicity let us first examine the case of the free scalar(Higgs) field; the action is then

$$\int d^4x\ dy\ \left[(\partial_A\Phi)^\dagger\left(\partial^A\Phi\right) - \mu^2\Phi^\dagger\Phi - k_\Phi\left(\partial_5\Phi\right)^\dagger\left(\partial_5\Phi\right)\right], \tag{3.1}$$

where we have allowed a standard bulk mass term only for this discussion. Performing the Kaluza-Klein(KK) decomposition as usual

$$\Phi = \sum_n \phi_n(x)\chi_n(y)\,, \tag{3.2}$$

and imposing the orbifold boundary conditions one obtains the usual eigenfunctions $\chi_n(y)\sim \cos q_n y$ for Z_2-even fields where $q_n^2 = m_n^2 - \mu^2$, with m_n being the KK mass. In addition, these wavefunctions also have the standard normalization $\int\ dy\ \chi_n(y)\chi_m(y) = \delta_{nm}$. However, the KK spectrum is now somewhat different than usual:

$$m_n^2 = \mu^2 + \frac{n^2}{R^2}(1+k_\Phi)\,, \tag{3.3}$$

where R is the compactification radius and n is an integer. The effect of the LV term is to rescale the KK excitation spectrum by some arbitrary amount. (Recall the we expect the dimensionless quantity k_Φ to be as large as order unity.) This is quite similar to the loop-induced radiative Higgs mass shift found in the case of UED induced by brane kinetic terms [4]. In that model the size of the contribution was logarithmically dependent on the cutoff but was under control numerically; here the rescaling occurs at the tree-level and is completely arbitrary. In order to insure a tachyon-free spectrum, i.e. , stability, it is clear that we must have $k_\Phi > -1$.

Next we turn to the gauge fields; when the corresponding gauge group is not spontaneously broken, e.g. , for the case of gluons in $\mathrm{SU}(3)_c$, the action is

$$\int d^4x\ dy\ \left[-\frac{1}{4}F_{AB}F^{AB} - \frac{\lambda_c}{4}\left(F_{\mu 5}F^{\mu 5} + F_{5\mu}F^{5\mu}\right)\right], \tag{3.4}$$

where color indices have been suppressed. The KK decomposition is most conveniently performed in the physical $g_5 = 0$ gauge:

$$g_\mu = \sum_n g_\mu^{(n)}(x) f_n(y)\,, \tag{3.5}$$

and produces the standard eigenfunctions $\sim \cos ny/R$ for Z_2-even fields normalized as usual. In a manner similar to the scalar case above, the KK masses are, however, now given by

$$m_{g_n}^2 = \frac{n^2}{R^2}(1+\lambda_c)\,, \tag{3.6}$$

where λ_c can be O(1). This spectrum shift is again similar to that induced by brane term radiative corrections in the UED model but here it can rescale the spectrum arbitrarily by a large amount. Since λ and k_Φ are completely unrelated, this rescaling of the KK spectra can be performed independently for these fields.

In the electroweak sector the KK decomposition is a bit more complex due to presence of symmetry breaking, the mixing among the neutral fields and the fact that the LV coefficients for the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ gauge groups, $\lambda_{W,B}$, respectively, can be numerically different. The case of the W KK tower is rather straightforward since the effect of symmetry breaking here is to generate a simple bulk mass term with the usual eigenfunctions; one obtains in standard notation

$$m_{W_n}^2 = \frac{1}{4}g^2v^2 + \frac{n^2}{R^2}(1+\lambda_W)\,, \tag{3.7}$$

as we might have expected. Note that we can adjust the W and gluon towers relative to each other in an arbitrary manner; in UED the ratio of these two, loop-induced shifts is completely fixed. For the neutral fields one obtains a level-by-level mixing between the KK excitations of the B and W_3 fields as in the case of UED. The elements of the symmetric KK mixing matrix at the n^{th} level are given by

$$M_{W_3W_3}^2 = \frac{1}{4}g^2v^2 + \frac{n^2}{R^2}(1+\lambda_W)$$

$$
\begin{aligned}
M^2_{W_3B} = M^2_{BW_3} &= \frac{1}{4} g g' v^2 \\
M^2_{BB} &= \frac{1}{4} g'^2 v^2 + \frac{n^2}{R^2}(1+\lambda_B)\,,
\end{aligned} \tag{3.8}
$$

which is somewhat similar to the case of UED. The corresponding level-dependent 'Weinberg-angle' is then given by

$$
\tan 2\theta_n = \frac{-2gg'v^2}{4(\lambda_B - \lambda_W)\frac{n^2}{R^2} + (g'^2 - g^2)v^2}\,. \tag{3.9}
$$

Since $\lambda_{B,W}$ are arbitrary, in principle O(1) parameters, this KK mass spectrum for these two neutral fields can be substantially different than obtained in UED.

The case for fermions proceeds in the standard fashion from the above action. Let us ignore $SU(2)_L \times U(1)_Y$ symmetry breaking and zero-mode mass generation for the moment; we will return to this issue below. The arbitrary 5d fermion field Ψ is decomposed into left- and right-handed pieces in the standard manner: $\Psi = P_L\Psi_L + P_R\Psi_R$ using the usual projection operators and then the KK decomposition is performed, i.e. , $\Psi_{L,R} = \sum_n \psi^{(n)}_{L,R} f^{(n)}_{L,R}(y)$ and we arrive at the the coupled equations

$$
\begin{aligned}
(1+k_\Psi)\partial_y f^{(n)}_L &= m_n f^{(n)}_R \\
-(1+k_\Psi)\partial_y f^{(n)}_R &= m_n f^{(n)}_L\,,
\end{aligned} \tag{3.10}
$$

so that the fermion masses are given by

$$
m^2_{\Psi_n} = (1+k_\Psi)^2 \frac{n^2}{R^2}\,. \tag{3.11}
$$

For doublet(singlet) fields we choose the left(right)-handed fermions to be Z_2-even to obtain the conventional SM structure. Note that the mass-squared of the fermion fields are quadratic in the LV correction whereas bosons experience a linear correction. In either case we again see that we can rescale the mass spectrum as we'd like since we can choose the LV coefficients arbitrarily. In particular, there is no reason, e.g. , for the left- and right-handed SM fermions to have KK towers that are in any way degenerate which can lead to new physics signatures as will be discussed below. Although KK-parity is still maintained at this point, clearly if one can rescale all of the masses of the KK excitations of the SM fields by arbitrary amounts it is no longer clear which state will be the lightest one in the spectrum. The identity of the LKKP dark matter candidate now depends on the values of the LV coefficients. We note that since we can rescale the fermion and boson spectrum as we'd like the possibility of confusion between the UED and SUSY scenarios at the LHC is now significantly increased.

The fact that the KK excitations of left-handed doublet and right-handed singlet fermions now have different tree-level masses directly leads to new phenomena. As a demonstration of this, consider for simplicity the toy model of 5d QED accompanied by LV. The KK towers of the of the left-handed and right-handed electron now having different masses will produce a signal for *parity violation* within a conventionally parity conserving

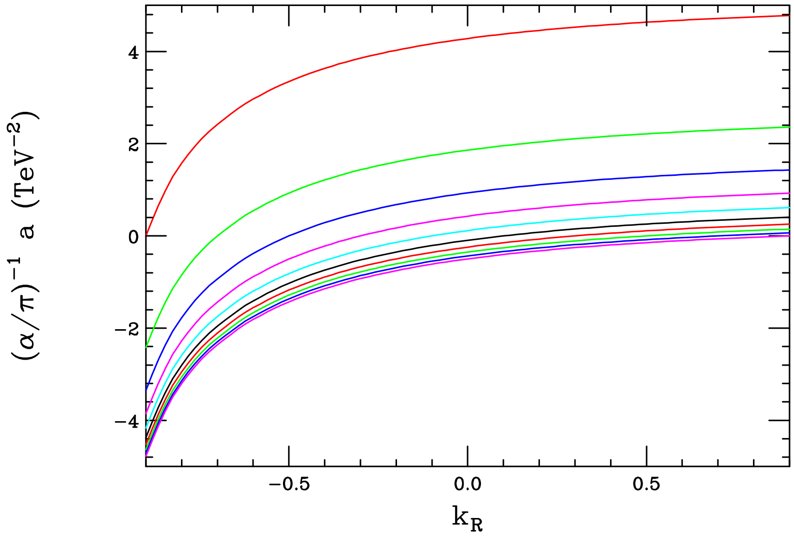

Figure 1: The anapole moment of the electron induced by LV in 5d QED assuming $\lambda_\gamma = 0$. From top to bottom the curves correspond to k_L =-0.9, -0.7,-0.5, etc

scenario as we will now demonstrate. If one considers the coupling between the (zero-mode) photon and left- and right-handed electrons one finds that at loop level a parity violating coupling will be generated, i.e. , the anapole moment [8], which corresponds to a tensor structure

$$< f|J_\mu^{em}|f >_{\rm anapole} = ieQ_f \bar{f}[q^2\gamma_\mu - \gamma\cdot q q_\mu]\gamma_5 F_3(q^2) f\,, \tag{3.12}$$

with $F_3(q^2)$ being the anapole moment form factor. $F_3(0) = a$ is then just the standard anapole moment which has dimensions $\sim R^2$. The existence of this coupling is directly related to the fact that the masses of the KK states inside the loop are different for the left- and right-handed towers; in obvious notation and summing over KK levels we find that

$$a \simeq \frac{\alpha}{\pi}\,\frac{\pi^2 R^2}{48}\int_0^1 dx \int_0^{1-x} dy \left[\frac{4(1-x)(1-y)+5xy}{1+\lambda_\gamma+[(1+k_R)^2-1-\lambda_\gamma](x+y)} - (R\to L)\right]. \tag{3.13}$$

Assuming that R^{-1} =500 GeV and $\lambda_\gamma = 0$ for purposes of demonstration we obtain the numerical results for a shown in figure 1; here we see that $|a| \sim \alpha\,\mathrm{TeV}^{-2}$, which to set the scale, is comparable in magnitude to the conventional SM contribution [8] induced by the parity-violating weak interaction. Clearly LV in 5d can lead to parity violating signatures in 4d in the absence of weak interactions.

Within the 5d UED scenario the analysis above also leads directly to a QCD color-anapole moment which also violates parity in 4d.

As we have just seen the introduction of 5d LV violating operators with O(1) coefficients allows us to modify the overall scales of the various gauge, scalar and left- and right-handed fermion KK spectra in UED in an independent fashion. Thus when such operators are present it is no longer clear that, e.g. , a neutral field will be the lightest state which is odd under KK-parity and we may lose our natural dark matter candidate. The situation is actually more severe than this as we will now see.

4. Influence of LV terms: KK-parity violation

The remaining term in the action that we have yet to examine is the Yukawa coupling between the fermion doublet and singlet fields and the Higgs boson that generates non-zero masses for the (would-be) zero-mode SM fermions. We can write this generically, dropping all generation labels, as:

$$\int d^4x\ dy\ \lambda_5 \bar{D} S \Phi + \text{h.c.}\,. \tag{4.1}$$

After rescaling by the field redefinitions employed above to rid ourselves of the unwanted 'CPT violating' LV terms this action becomes

$$\int d^4x\ dy\ \lambda_5 e^{i(\Sigma_\Phi+\Sigma_S-\Sigma_D)y} \bar{D} S \Phi + \text{h.c.}\,, \tag{4.2}$$

so that a position-dependent phase has crept into the Yukawa part of the action. It is important to recall that the quantities Σ_i are Z_2-odd, i.e. , they flip their sign at the origin. To probe the influence of this term let us first extract out the all zero-mode piece and perform the y-integration. Recall that zero-mode wave functions are flat $= 1/\sqrt{2\pi R}$; we obtain the 4d integrand

$$\frac{\lambda_5}{\sqrt{2\pi R}}\ \frac{v+H}{\sqrt{2}}\ e^{i\sigma\pi R/2}\ \frac{\sin(\sigma\pi R/2)}{\sigma\pi R/2}\,, \tag{4.3}$$

where $\sigma = \Sigma_\Phi + \Sigma_S - \Sigma_D$ and H is the usual SM Higgs field. The SM 4d Yukawa coupling can then be identified as

$$\lambda_4 = \frac{\lambda_5}{\sqrt{2\pi R}}\ e^{i\sigma\pi R/2}\ \frac{\sin(\sigma\pi R/2)}{\sigma\pi R/2}\,. \tag{4.4}$$

Apart from the overall phase factor the last term can substantially rescale the size of the Yukawa coupling depending on the value of σR and may lead to some interesting phenomenology.

Something even more interesting results when we do not project into the all zero-mode state. Due to the additional y-dependent phase these Yukawa terms can *violate* KK-parity causing, e.g. , a destabilization of the LKKP. Recall that in the usual UED model KK-parity is preserved to all orders in perturbation theory. To see this effect it is useful to examine the mixing between the would-be zero mode fermion and the Z_2-even members of the KK tower; this corresponds to the off-diagonal sub-matrix linking, e.g. , the zero-mode doublet field with a KK singlet state. This calculation is straightforward and, in terms of the 4d Yukawa coupling λ_4 is given by

$$\frac{\lambda_4 v}{2}\left[e^{in\pi/2}\ \frac{\sigma R}{\sigma R+n}\ \frac{\sin((\sigma R+n)\pi/2)}{\sin(\sigma\pi R/2)}\right] + (n \to -n)\,, \tag{4.5}$$

which corresponds to the $0n$ element of the KK mass sub-matrix, M_{0n}, and is seen to be proportional to the SM zero mode mass, $M_{0n} = \delta_{0n} m_f$. (It is important to note that here the symbol δ_{0n} does *not* denote the Kronecker delta.) Clearly, such terms can only be significant if σR is O(1) but this might be expected. Furthermore, one finds that all of the sub-matrix elements of this type, $M_{nm} = m_f \delta_{nm}$, are in general found to be non-zero with a mass scale set by the conventional SM fermion mass, i.e. , with δ_{nm} values generally of order unity and controlled by the values of n, m and σR. This is unlike the case of UED where the mixing between the D and S fermion fields takes place level by level; here there is also a potentially significant mixing between the various KK levels. However, light fermions, such as the electron, experience little direct KK-parity violation through such mixing whereas for heavy fields, like the top quark, this violation can be quite significant for $R^{-1} \sim 1\,\text{TeV}$ or less. The removal of the 'CPT-violating' LV terms in the original action via field redefinitions has thus resulted in the breakdown of KK-parity conservation. We note that KK-parity violation at some level might also occur if UED is extended higher dimensions to include gravitational effects [9].

This violation of KK-parity is quantifiable at the tree level by estimating the lifetime of the LKKP. To get an order of magnitude estimate we perform the calculation in the mass insertion approximation and assume that as usual the first KK photon excitation with mass M is the LKKP. The process proceeds via $\gamma_1 \to \bar{f}_{L0} f_{L1} + \text{h.c.} \to \bar{f}_{L0} f_{L0} + (L \to R)$, where the second step arises from mixing. We obtain

$$\Gamma = N_c Q_f^2 (2Re\ \delta_{01})^2\ \frac{\alpha M}{6}\ (1 - 4m_f^2/M^2)^{1/2} \left[(g_L^2 + g_R^2) \left(1 - \frac{m_f^2}{M^2} \right) + 6 g_L g_R \frac{m_f^2}{M^2} \right] , \quad (4.6)$$

where m_f is the would-be zero mode mass, $M = M_{\gamma_1}$,

$$g_{L,R} = \frac{m_f^2}{(m_{L,R}^2 - m_f^2)} , \quad (4.7)$$

with $m_{L(R)}$ being the mass of the first KK excitation of the doublet (singlet) field $f_{L(R)}$ and δ_{01} is the dimensionless mixing parameter defined above.

Here we see again an example of induced parity violation in that the two couplings are equal, $g_L = g_R$, only when the fermion KK excitation masses are the same. Note that as expected this decay is very highly suppressed for light fermions, i.e. , decays to heavy fermions such as top quarks, will be by far dominant. To get an idea of the size of this suppression, we take $m_L = m_R = M$ and $2Re(\delta_{01}) = 1$ so that

$$\Gamma = \frac{N_c}{3} Q_f^2 \alpha\ M\ F(x) , \quad (4.8)$$

with $x = m_f/M$; the function $F(x)$ is shown in figure 2. As we expected, except for the closure of phase space $F(x)$ is larger the closer x is to $1/2$; decays to first generation fields is thus seen to be highly suppressed. Although the expression above might correspond to a very narrow width by usual collider standards, for any reasonable range of parameters the lifetime of the LKKP is quite short in comparison to the age of the universe. Clearly, if some other particle is actually the LKKP, the analogous calculation can be performed obtaining qualitatively similar results.

Another way to observe the violation of KK-parity is through loop-induced mixing among different gauge boson KK levels. This mixing is induced by the insertion of off-diagonal fermion mass matrix elements into vacuum polarization graphs connecting gauge

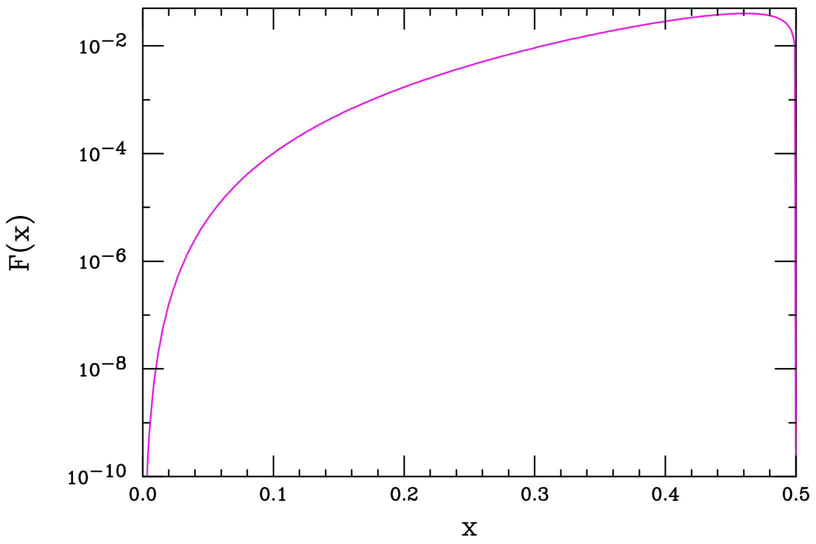

Figure 2: The function F(x) as defined in the text.

fields with different KK number. In the 5d QED example this corresponds to a process $\gamma_n \to \bar{f}_n f_0 + \text{h.c.} \to \bar{f}_m f_0 + \text{h.c.} \to \gamma_m$ where the intermediate step occurs due to Yukawa induced fermion mixing. Using the notation above, mass mixing arising from this process in the photon tower mass matrix induced by a single fermion flavor is given by

$$\delta M^2_{mn} \simeq N_c Q_f^2 \, \frac{\alpha}{\pi} \, 2Re(\delta_{mn}) \, m_f^2 \, G\left(\frac{m_f^2}{m^2_{S_n}}, \frac{m_f^2}{m^2_{D_m}}\right) + (n \to m)\,, \tag{4.9}$$

with m_{D_m,S_n} being the masses of the KK fermions in the loop and G is an order one loop function. Here we again see that the dominant contribution arises from the most massive SM fermion sector as we might have expected. Clearly with δ_{nm}'s of order unity a summation over all possible fermions in the loop can lead to small yet significant mixing in the gauge boson mass matrix. This result easily generalizes to the cases of the W, Z and gluon KK towers where gauge KK mass eigenstates will now no longer have a definite KK-parity. A similar mixing will occur among Higgs and Goldstone KK levels.

One of the other effects of KK-parity conservation in the UED model is the inability to singly produce states which are KK-parity odd at colliders, e.g. , the lightest KK gauge boson excitations. The violation of KK-parity induced by Yukawa interactions leads to modifications of this conventional result though the corresponding cross sections are not necessarily large. This can be seen by the fact that the widths of the KK-odd gauge bosons into zero modes of the first two generations is quite small.

In this section we have seen that the elimination of the 5d analogs of the 4d 'CPT violating' operators by field redefinitions induces potentially large violations of KK-parity. We observed that the size of this violation an any given SM fermion sector is correlated with the known size of the would-be zero mode masses. As a result UED loses its dark matter candidate.

JHEP09(2005)036

5. Summary and conclusions

In this paper we have initiated a study of the influence of explicit Lorentz violation within the context of the 5d SM where all fields are in the bulk, i.e. , the Universal Extra Dimensions scenario. To perform this analysis we extended the 'conventional' 4d model of Colladay and Kostelecky to 5d and searched for a subset of operators that can leave 4d Lorentz invariance untouched while breaking it in 5d. Two of these operators, the 5d analogs of those that violate CPT in 4d, can be (almost) removed from the action through a set of field redefinitions for fermions and scalars. One obvious result of this field redefinition is to induce a negative mass square term in the Higgs potential which may be the source of electroweak symmetry breaking. In addition, the natural scale of the induced vev would be $\sim R^{-1}$ thus linking the scale of electroweak symmetry breaking with the size of the extra dimension. The remaining LV operators lead to alterations of the various gauge, Higgs and fermionic kinetic terms and independently rescale their associated KK spectra which can increase the possible confusion of UED and SUSY at the LHC. Since, e.g. , the masses of KK excitations of the left- and right-handed SM fermions need no longer be equal this induces, at loop order, parity-violating effects in previously parity-conserving parts of the SM, i.e. , QED and QCD. Furthermore, we have shown that the the field redefinitions used to eliminate the 5d analogs of the 4d CPT violating terms make an important change in the nature of the Yukawa couplings responsible for generating the would-be zero-mode fermion masses. Due to an additional fifth co-ordinate-dependent phase, fermion mass terms are generated that produce mixing among all of the various KK levels thus violating KK-parity. This leads to a destabilization of the lightest KK-odd particle which is the usual dark matter candidate in UED. In addition these terms were shown to induce mixing between the various gauge KK levels at one-loop.

As we have seen the presence of LV terms in the 5d UED scenario can lead to substantial modifications from the conventional expectations. Hopefully signals for extra dimensions will be found at future colliders.

Acknowledgments

The author would like to thank J.Hewett and B. Lillie for discussions related to this work.

A. LV in 6d

It is interesting to consider what happens when LV is extended to 6d and compactified on an orthogonal torus with radii $R_{5,6}$. Although a detailed study lies outside the scope of the current work we would like to give some flavor here by considering for simplicity the LV 6d scalar action. From the analysis above, this is given by

$$\int d^4x\; dx_5\; dx_6 \left[(D_A\Phi)^\dagger \left(D^A\Phi\right) - V\left(\Phi^\dagger\Phi\right) - k_{ij}\left(D_i\Phi\right)^\dagger \left(D_j\Phi\right) + ih_i\left(\Phi^\dagger D_i\Phi - \Phi D_i\Phi^\dagger\right)\right], \tag{A.1}$$

where we take $k_{i,j}$, $i,j = 5,6$, to be real and symmetric; summation over these indices when repeated is implied. Since we will be concentrating for simplicity on the pure scalar sector in our discussion below, we have ignored the possibility of new LV interaction terms that may be present in 6d which are absent in 5d. As in 5d, the 'CPT-violating' terms can be eliminated by a field redefinition:

$$\Phi \to e^{i\Sigma_i x_i}\Phi\,, \tag{A.2}$$

where

$$\Sigma_5 = \frac{-h_5(1+k_{66}) + h_6 k_{56}}{(1+k_{55})(1+k_{66}) - k_{56}^2}\,, \tag{A.3}$$

and Σ_6 can be obtained by interchanging 5 and 6 in the expression above. As in 5d this field redefinition adds a new, likely negative term to the scalar potential:

$$-\frac{h_5^2(1+k_{66}) + h_6^2(1+k_{55}) + 2h_5 h_6 k_{56}}{(1+k_{55})(1+k_{66}) - k_{56}^2}\,\Phi^\dagger\Phi\,. \tag{A.4}$$

So far this is a rather straightforward extension of 5d; something new happens when we perform the usual KK decomposition

$$\Phi(x_\mu, x_i) = \sum_{n,m} \phi_{n,m}(x_\mu)\chi_{n,m}(x_i)\,. \tag{A.5}$$

Through the usual manipulations we are led to the equation of motion for χ which we can write for free scalars as

$$\partial_i\left[h^{ij}\partial_j\chi\right] - m_{n,m}^2\chi = 0\,, \tag{A.6}$$

where the symmetric object h_{ij} acts as a flat, constant 'metric' in the $x_5 - x_6$ space with elements $h_{55} = 1 + k_{55}$, $h_{66} = 1 + k_{66}$ and $h_{56} = k_{56}$. These satisfy $h_{il}h^{lj} = \delta_i^j$ and thus h^{ij} are the elements of the inverse matrix h^{-1}. In the $x_5 - x_6$ co-ordinate basis this equation is not generally separable; however, the metric can be diagonalized through a suitable $x_5 - x_6$ rotation to the basis $x_\pm$:

$$\begin{aligned} x_+ &= x_5\cos\theta + x_6\sin\theta \\ x_- &= x_6\cos\theta - x_5\sin\theta\,, \end{aligned} \tag{A.7}$$

with angle θ given by

$$\tan 2\theta = \frac{2k_{56}}{k_{55} - k_{66}}\,, \tag{A.8}$$

so that the now separable equation of motion for χ becomes

$$\lambda_+^{-1}\partial_+^2\chi + \lambda_-^{-1}\partial_-^2\chi + m_{n,m}^2\chi = 0\,, \tag{A.9}$$

with $\lambda_\pm$ given by

$$\lambda_\pm = 1 + \frac{k_{55}+k_{66}}{2} \pm \frac{1}{2}\left[(k_{55}-k_{66})^2 + 4k_{56}^2\right]^{1/2}. \tag{A.10}$$

Note that although our metric is constant, rotations no longer commute with it. The fact that there is a 'preferred' frame where the 'metric' is diagonal is the result of LV here manifest as the loss of $x_5 - x_6$ rotational invariance. We can now express χ as $\chi_{n,m} = f_n(x_+)g_m(x_-)$ in this preferred basis.

Although we have switched to the co-ordinates $x_\pm$, the boundary conditions will most likely be expressed in the $x_{5,6}$ basis. Here, for example, we consider the most simple case where we have invariance under the typical periodic conditions: $x_{5,6} \to x_{5,6} + 2\pi R_{5,6}$, so that one can write $\chi = \exp in_i x_i/R_i = \exp i[a_+x_+ + a_-x_-]$ and thus

$$m^2_{n_5,n_6} = \frac{a_+^2}{\lambda_+} + \frac{a_-^2}{\lambda_-}\,, \tag{A.11}$$

where

$$\begin{aligned} a_+ &= \frac{n_6}{R_6}\cos\theta - \frac{n_5}{R_5}\sin\theta \\ a_- &= \frac{n_5}{R_5}\cos\theta + \frac{n_6}{R_6}\sin\theta\,. \end{aligned} \tag{A.12}$$

Note that in this simple case, the KK mass eigenvalue equation could have been obtained without making the co-ordinate transformation above since the eigenfunctions are simple exponentials. Straightforward algebra yields

$$m^2_{n_5,n_6} = \left[(1+k_{55})(1+k_{66}) + k_{56}^2\right]^{-1}\left((1+k_{66})\frac{n_5^2}{R_5^2} + (1+k_{55})\frac{n_6^2}{R_6^2} - 2k_{56}\frac{n_5n_6}{R_5R_6}\right). \tag{A.13}$$

This eigenvalue equation for the KK masses is remarkably similar to that obtained by Dienes [10] who consider tori with shift angles and shape moduli in 6d. Instead of the simple KK spectrum rescaling that we observed for LV in 5d, in 6d the KK spectrum is significantly skewed and distorted compared to conventional expectations. The shift angle of Dienes in our case arises from LV and the existence of the preferred frame.

A more detailed discussion of LV in 6d will be given elsewhere.

References

[1] For a couple of recent reviews, see for example D. Mattingly, *Modern tests of Lorentz invariance*, gr-qc/0502097;
H. Vucetich, *Testing Lorentz invariance violation in quantum gravity theories*, gr-qc/0502093;
T. Jacobson, S. Liberati and D. Mattingly, *Lorentz violation at high energy: concepts, phenomena and astrophysical constraints*, astro-ph/0505267;
For early work on this subject in the context of strings, see V.A. Kostelecky and S. Samuel, *Spontaneous breaking of lorentz symmetry in string theory,Phys. Rev.* **D 39** (1989) 683;
Gravitational phenomenology in higher dimensional theories and strings, *Phys. Rev.* **D 40** (1989) 1886;
V. A. Kostelecky and R. Potting, *CPT and strings*, *Nucl. Phys.* **B 359** (1991) 545.

[2] See, for example, M.L. Graesser, A. Jenkins and M.B. Wise, *Spontaneous Lorentz violation and the long-range gravitational preferred-frame effect*, *Phys. Lett.* **B 613** (2005) 5 [hep-th/0501223];

JHEP09(2005)036

R. Bluhm and V.A. Kostelecky, *Spontaneous Lorentz violation, nambu-goldstone modes and gravity*, *Phys. Rev.* **D 71** (2005) 065008 [hep-th/0412320];
N. Arkani-Hamed, H.-C. Cheng, M. Luty and J. Thaler, *Universal dynamics of spontaneous Lorentz violation and a new spin-dependent inverse-square law force*, *JHEP* **07** (2005) 029 [hep-ph/0407034];
L.P. Colatto, A.L.A. Penna and W.C. Santos, *Charged tensor matter fields and Lorentz symmetry violation via spontaneous symmetry breaking*, *Eur. Phys. J.* **C 36** (2004) 79 [hep-th/0310220];
J.W. Moffat, *Spontaneous violation of Lorentz invariance and ultra-high energy cosmic rays*, *Int. J. Mod. Phys.* **D12**20031279 [hep-th/0211167];
D. Colladay, *Spontaneous violation of Lorentz and CPT symmetry*, hep-ph/0103021.

[3] D. Colladay and V.A. Kostelecky, *Lorentz-violating extension of the standard model*, *Phys. Rev.* **D 58** (1998) 116002 [hep-ph/9809521]; *CPT violation and the standard model*, *Phys. Rev.* **D 55** (1997) 6760 [hep-th/9703464].

[4] T. Appelquist, H.-C. Cheng and B.A. Dobrescu, *Bounds on universal extra dimensions*, *Phys. Rev.* **D 64** (2001) 035002 [hep-ph/0012100];
H.-C. Cheng, K.T. Matchev and M. Schmaltz, *Bosonic supersymmetry? Getting fooled at the LHC*, *Phys. Rev.* **D 66** (2002) 056006 [hep-ph/0205314]; *Radiative corrections to Kaluza-Klein masses*, *Phys. Rev.* **D 66** (2002) 036005 [hep-ph/0204342];
T.G. Rizzo, *Probes of universal extra dimensions at colliders*, *Phys. Rev.* **D 64** (2001) 095010 [hep-ph/0106336].

[5] V.A. Kostelecky, *Gravity, Lorentz violation and the standard model*, *Phys. Rev.* **D 69** (2004) 105009 [hep-th/0312310].

[6] V.A. Kostelecky and R. Lehnert, *Stability, causality and Lorentz and CPT violation*, *Phys. Rev.* **D 63** (2001) 065008 [hep-th/0012060].

[7] M. Carena, T.M.P. Tait and C.E.M. Wagner, *Branes and orbifolds are opaque*, *Acta Phys. Polon.* **B33** (2002) 2355 [hep-ph/0207056]; *Opaque branes in warped backgrounds*, *Phys. Rev.* **D 67** (2003) 096006 [hep-ph/0212307];
F. del Aguila, M. Perez-Victoria and J. Santiago, *Physics of brane kinetic terms*, *Acta Phys. Polon.* **34** (2003) 5511 [hep-ph/0310353]; *Bulk fields with brane terms*, *Eur. Phys. J.* **C 33** (2004) S773 [hep-ph/0310352]; *Some consequences of brane kinetic terms for bulk fermions*, [hep-ph/0305119]; *Bulk fields with general brane kinetic terms*, *JHEP* **0302** (2003) 051 [hep-ph/0302023];
F. del Aguila, M. Perez-Victoria and J. Santiago, *Physics of brane kinetic terms*, *Acta Phys. Polon.* **B34** (2003) 5511 [hep-ph/0310353];
H. Davoudiasl, J.L. Hewett and T.G. Rizzo, *Brane localized curvature for warped gravitons*, *JHEP* **08** (2003) 034 [hep-ph/0305086]; *Brane localized kinetic terms in the Randall-Sundrum model*, *Phys. Rev.* **D 68** (2003) 045002 [hep-ph/0212279].

[8] Anapole moments for SM fermions have been considered by many authors see, for example L.G. Cabral-Rosetti, M. Moreno and A. Rosado, *Dirac neutrino anapole moment*, hep-ph/0206083;
A. Rosado, *Physical electroweak anapole moment for the neutrino*, *Phys. Rev.* **D 61** (2000) 013001;
S. Kulkarni, C.S. Warke and Y.K. Gambhir, *Relativistic mean field approach to anapole moment: atomic parity violating hyperfine transitions*, *Phys. Rev.* **C 52** (1995) 1047;

A. Gongora and R.G. Stuart, *The Charge radius and anapole moment of a free fermion*, *Z. Physik* **C 55** (1992) 101;
M.J. Musolf and B.R. Holstein, *Observability of the anapole moment and neutrino charge radius*, *Phys. Rev.* **D 43** (1991) 2956;
H. Czyz and M. Zralek, *The anapole moment and singlet fields*, *Can. J. Phys.* **66** (1988) 384;
H. Czyz, K. Kolodziej, M. Zralek and P. Khristova, *Is the anapole moment a physical observable?*, *Can. J. Phys.* **66** (1988) 132;
M. Abak and C. Aydin, *Calculation of the anapole moment of the neutrino*, *Europhys. Lett.* **4** (1987) 881;
N. Dombey and A.D. Kennedy, *Calculation of the electron anapole moment*, *Phys. Lett.* **B 91** (1980) 428.

[9] A. De Rujula, A. Donini, M.B. Gavela and S. Rigolin, *Fat brane phenomena*, *Phys. Lett.* **B 482** (2000) 195 [hep-ph/0001335];
C. Macesanu, C.D. McMullen and S. Nandi, *New signal for universal extra dimensions*, *Phys. Lett.* **B 546** (2002) 253 [hep-ph/0207269]; *Collider implications of universal extra dimensions*, *Phys. Rev.* **D 66** (2002) 015009 [hep-ph/0201300];
see also the last paper in ref. [4] C. Macesanu, C.D. McMullen and S. Nandi, *New signal for universal extra dimensions*, *Phys. Lett.* **B 546** (2002) 253 [hep-ph/0207269].

[10] K.R. Dienes, *Shape versus volume: making large flat extra dimensions invisible*, *Phys. Rev. Lett.* **88** (2002) 011601 [hep-ph/0108115].

JHEP09(2005)036